Magnetic Stars, 2011, pp. 61 – 68

Theory of Polarized Radiation from Accretion Disks: the Modern State of the Problem

Gnedin Yu. N., Piotrovich M. Yu., Natsvlishvili T. M., Silant'ev N. A.

Central Astronomical Observatory at Pulkovo, Saint–Petersburg, Russia

Abstract. We present the method that allows to estimate the magnetic field strength in the accretion disks surrounding stars of various types. Polarization originates as a result of scattering of light emitted in the magnetized accretion disk. The main feature of polarization produced is the wavelength dependence of the polarization degree and positional angle. It means that the polarization spectrum of scattered radiation depends strongly on the accretion disk model. This phenomenon allows to test various models of accretion disks around stellar objects. We obtain the values of polarization parameters of various models of optically thick magnetized accretion disks.

1 Introduction

One of the research areas which has recently shown great growth is the study of stellar and active galaxy nucleus magnetic fields. The most direct method of measurement of cosmic magnetic fields is based on the famous Zeeman effect of atomic spectral lines which was discovered by Zeeman in 1896. The classical Zeeman splitting of atomic spectral lines is the usual situation in ordinary laboratory conditions as it requires quite a small and moderate field strength: $B \leq 10^3$ G. A list of modern methods of cosmic magnetic field measurements is presented in Table 1.

Below there are some comments to Table 1. Another popular method of magnetic field measurements is based on the Hanle effect. This method is based on the mechanism of coherent resonance scattering in the spectral line which does not depend on the Doppler effect. In this effect the quantum interference between the magnetic sublevels of the upper resonance level takes place. As a result of this interference, the polarization of resonance scattering is radically changed. The change is connected with two effects: (a) depolarization of resonance scattering and (b) rotation of the linear polarization plane, i. e. the analogue of the classical Faraday rotation of the polarization plane.

To observe the stellar magnetic field by the Zeeman method one needs to choose some suitable spectral lines with distant components of multiplets. The problem becomes quite complex because the components are significantly broadened due to the Doppler effect linked with thermal motion of atoms. Besides, the existence of opposite polarity magnetic fields on the stellar surface complicates the problem since the contributions from the fields of various directions cancel one another. Hence, the Zeeman method does not work well for the determination of magnetic field strengths in hot stars such as, for example, Be and WR stars. For compact stars (magnetic white dwarfs, neutron stars, accreting disks around black holes) with quite a strong magnetic field the Zeeman method is also inapplicable.

Many stars have an envelope consisting of magnetized plasma. For many astrophysical objects (stars, black holes, supermassive black holes in active galaxy nuclei) the accretion disk is an important element. The radiation of these astrophysical objects acquires linear polarization as a result of

Table 1: A list of modern methods of cosmic magnetic field measurements

Traditional Methods
1. Zeeman spectropolarimetry.
2. Hanle effect.
3. Circular broadband polarimetry.
4. Cyclotron spectroscopy: white dwarfs, neutron stars.
5. Faraday rotation measure (RM) of position angles — in radio astronomy.
6. Spectrum and polarization of synchrotron radiation.
New Methods
1. Proton cyclotron spectroscopy: Neutron Stars — X–ray proton cyclotron lines.
2. Magnetic fields due to Faraday rotation on the electron scattering free path length.
3. Synchrotron radiation with synchrotron self-absorption.

scattering on the electrons in the envelope and accretion disk. This scattered radiation undergoes Faraday rotation by propagation in the magnetized plasma of the accretion disk and the envelope. The angle of the Faraday rotation Ψ is determined by the expression (Gnedin & Silant'ev, 1997):

$$\Psi = 0.4 \left(\frac{\lambda}{1\mu m}\right)^2 \left(\frac{B}{1G}\right) \tau \cos\theta = \frac{1}{2}\delta\tau\cos\theta, \tag{1}$$

where λ is the wavelength of radiation and θ is the angle between the directions of the magnetic field $\vec{B}$ and the line of sight $\vec{n}$. According to the Eq. (1) the Faraday dimensionless depolarization parameter δ takes a simple form:

$$\delta = 0.8\lambda^2(\mu m)B(G) \tag{2}$$

2 Linear Polarization of Light in Stellar Atmospheres with Magnetic Fields

There are numerous data about the existence of magnetic fields in stellar atmospheres. Let us discuss qualitatively the polarization of light in magnetized atmospheres with Faraday rotation. First of all, let us remind the known results concerning the usual non-magnetized atmospheres where the scattering on free electrons gives rise to some linear polarization of the outgoing radiation. For the non-absorbing atmosphere, as it is well known (Chandrasekhar, 1950; Sobolev, 1963), the radiation has a maximal magnitude of polarization of 11.7% when the direction of the line of sight $\vec{n}$ is perpendicular to the normal $\vec{N}$ to the surface of the atmosphere. This polarization is due to the non–isotropic character of the intensity I of radiation near the atmosphere's surface and arises mostly as a result of the last scattering of light before escaping the atmosphere. The same effect takes place also in the presence of a magnetic field. The light scatters on free electrons and the scattered partly linearly polarized waves undergo the action of the Faraday rotation. As the outgoing radiation comes from various optical depths, it has very different angles of rotation. This effect produces depolarization of the outgoing radiation. Faraday rotation does not occur when the radiation propagates perpendicular to the magnetic field direction. Near this direction there exists a zone where $\delta\cos\theta \ll 1$, and in this zone the radiation conserves its polarization as in the case of the absence of magnetic field.

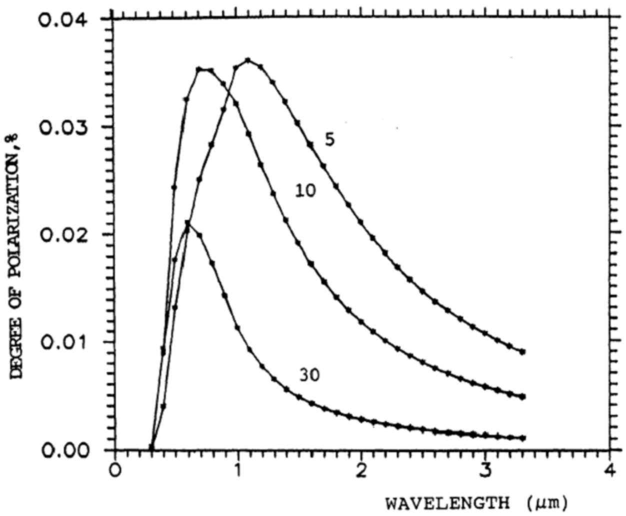

Figure 1: The ratio of the Stokes parameters for the model of the atmosphere with the effective temperature $T_e = 1.5 \times 10^4$ K and $\log g = 2$ (Kurucz et al., 1974)

Figure 1 represents the ratio of the Stokes parameters for the model of the atmosphere with the effective temperature $T_e = 1.5 \times 10^4$ K and $\log g = 2$ (Kurucz et al., 1974). The values of the opacity were taken from Allen (1973). For small magnetic field strengths the predominant oscillations of the electric field of the radiation occur perpendicular to the dipole axis. The maximum magnitude of the polarization degree corresponds to the values $\delta(1 - q_\lambda) \approx 1$, where q_λ is the dimensionless absorption coefficient (Silant'ev, 1993).

The considered models with dipole magnetic field give the upper limit for the integral polarization from magnetized stellar atmospheres in the continuum. Due to high sensitivity of the Faraday rotation to the value of the magnetic field, the observable polarization exists only for small magnetic fields with $B < 200$ G. For the magnetic fields $B > 200$ G, one can find that the polarization degree is close to zero. On the other hand, the Sun shows that the magnetic fields in the stellar atmospheres are distributed not so regularly as the dipole field. In reality, the magnetic field distribution looks like some kind of magnetic spots. This means that if the global magnetic field is greater that 200 G, then one may neglect integral polarization and consider that total polarization is due to radiation from the parts of stellar atmosphere beyond the spots.

The value of this polarization has been calculated by Silant'ev (1993). He obtained the following formula for polarization:

$$\frac{F_Q}{F_I} = -\sin 2\varphi_0 \times 0.393\%(1 - q_\lambda)\frac{1 - 1.372A_\lambda}{1 + (2/3)A_\lambda}. \quad (3)$$

In this case the spot is considered as some equatorial bar with the width $R_S \sin\varphi_0$, where R_S is the spot radius and A_λ is the tabulated coefficient of darkening to the edge of an atmosphere. It is interesting that the maximal polarization corresponds to $\varphi_0 = 45°$.

3 Integral Polarization from the Accretion Disk

The accretion disk is observed as a whole. Therefore, the Stokes parameters of radiation U and Q must be averaged in azimuth and the observed values for the degree of polarization and position angle are derived. This procedure was made by Silant'ev et al. (2009). In a result, the detailed description of the behaviour of these parameters was presented in this paper. The degree of polarization P and the polarization position angle χ for such a disk can be expressed in the analytical form:

$$P_l(\vec{B},\mu) = \frac{P_l(\mu)}{[1+2(a^2+b^2)+(a^2-b^2)^2]^{1/4}} \tag{4}$$

$$\tan 2\chi = \frac{U}{Q} = \frac{2a}{(P(\mu)/P_l(\vec{B},\mu))^2+1+b^2-a^2} \tag{5}$$

Here, μ is the cosine of the angle i between the directions of the line of sight and the normal to the disk $\vec{N}$. The value $P(\mu)$ is the degree of polarization in the case of the classical Milne problem, i.e. in the case of the scattering in non-magnetized, conservative atmosphere. $P_l(\mu)=11.7\,\%$ for $\mu=0$ (Chandrasekhar, 1950). The polarization angle $\chi=0$ corresponds to the oscillations of the electric vector of the wave parallel to the plane of the accretion disk. We consider here only the conservative atmospheres, which are usually adopted in various accretion disk models. The dimensionless depolarization parameters a and b describe the effect of the Faraday depolarization of radiation:

$$a = 0.8\lambda^2 B_{\|}\mu = \delta\|\mu b = 0.8\lambda^2 B_{\perp}\sqrt{1-\mu^2} = \delta\perp\sqrt{1-\mu^2}. \tag{6}$$

Here, $B_{\|}$ and $B_{\perp}=\sqrt{B_r^2+B_\varphi^2}$ are the components of the magnetic field along and perpendicular to the normal to the disk, respectively. The parameter $\delta=0.8\lambda^2 B$ is numerically equal to the Faraday rotation angle at the Thomson optical depth $\tau=2$ if the polarized radiation propagates along the magnetic field.

For particular cases of pure normal ($\delta_{\perp}=0$) and pure perpendicular ($\delta_{\|}=0$) magnetic fields, Eqs. (4) and (5) give the following relations:

$$P_l(\vec{B},\mu) = \frac{P_l(\mu)}{\sqrt{1+\delta_{\|}^2\mu^2}};\quad \tan 2\chi = \delta_{\|}\mu \tag{7}$$

$$P_l(\vec{B},\mu) = \frac{P_l(\mu)}{\sqrt{1+\delta_{\perp}^2(1-\mu^2)}};\quad \chi = 0 \tag{8}$$

Figure 2 presents the results of calculations (Dolginov et al., 1995) of polarization of radiation escaping from the plane parallel accretion disk atmosphere with a normal magnetic field. The numbers on the curves denote the values of the depolarization parameter $\delta_{\|}$. It can be seen that the degree of polarization has a peak-like form near the direction perpendicular to the magnetic field direction. The width of the polarization distribution decreases rapidly with an increasing magnetic field strength. The maximum of polarization is equal to 9.14 % instead of the classical 11.71 % value corresponding to multiple scattering without a magnetic field.

If the magnetic field $\vec{B}$ has an arbitrary direction, then instead of the peak value 9.14 % we have a smaller value (Gnedin & Silant'ev, 1997):

$$\frac{Q(\vec{n}\perp\vec{B})}{I(\mu)} = \frac{1-\mu^2}{3-\mu^2}\left[H(\mu)-\frac{3}{2}(H_2+\mu H_1)\right]/H(\mu) \tag{9}$$

where $H(\mu)$ is the well known H function of Chandrasekhar (1950) describing the angular dependence of outgoing radiation for Rayleigh scattering, $H_1=1.194$, $H_2=0.8494$.

There are three basic regions of magnetic fields in the nearest vicinity of a star: (a) accretion disk, (b) jet, (c) outflow, for example, stellar wind.

The accretion flow and the magnetic field of this flow have an important role in stellar evolution. The rotationally supported stellar disk is threaded by open magnetic field lines (Krasnopolsky &

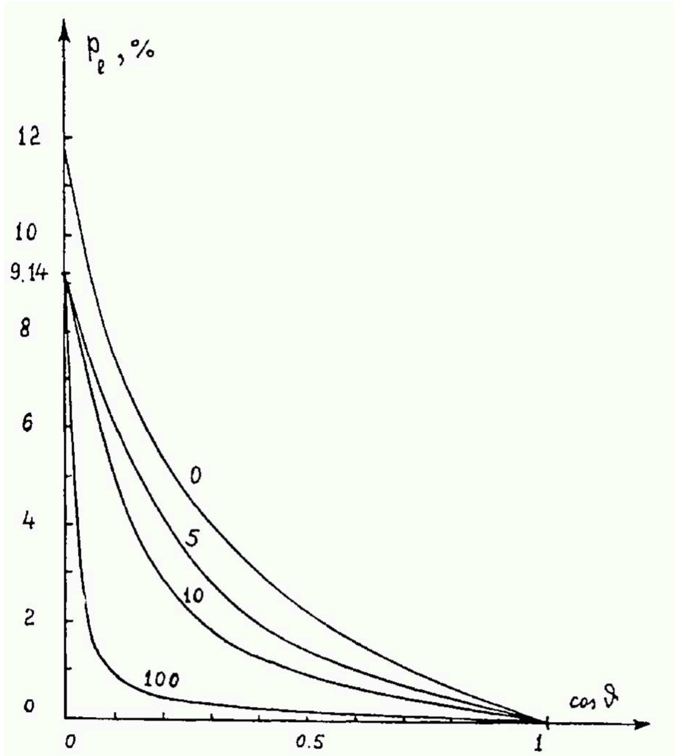

Figure 2: The results of calculations (Dolginov et al., 1995) of polarization of the radiation escaping from the plane parallel accretion disk atmosphere with a normal magnetic field.

Königl, 2002). At least a part of the angular momentum removed from the accreted matter is transported vertically by a large–scale field (rather than radially by viscous stresses) and deposited in the outflow. In a steady state, radial advection and azimuthal shearing of the disk magnetic field are balanced by magnetic diffusivity. The magnetic field plays an important role. It can be the source of viscosity, heat the disk corona and cause the flares, producing variability. The magnetic field can be also the source of the synchrotron radiation.

Now we consider the accretion disks around young stellar objects. Rotationally–supported circumstellar disks in the protostellar systems evidently originate in the collapse of self–gravitating, rotating molecular cloud cores. The cores are threaded by open interstellar magnetic field lines that are dragged inward once the dynamical collapse is initiated.

Young low–mass stars that accrete the material through their circumstellar disks can exhibit a dramatic increase in brightness in the optical range. For the advection–dominated accretion disks the basic equations have the form (Narayan & Li, 1994; Medvedev, 2000; Pariev et al., 2003):

$$B = 7\left(\frac{\dot{M}_{ac}}{10^{-7}M_\odot/\mathrm{yr}}\right)^{1/2}\left(\frac{M_S}{M_\odot}\right)^{1/4}\left(\frac{R}{0.1\mathrm{AU}}\right)^{-5/4}\ \mathrm{G}$$

$$T_e = 10^4\left(\frac{M_S}{M_\odot}\right)\left(\frac{R}{0.1\mathrm{AU}}\right)^{-1}\ \mathrm{K} \qquad (10)$$

$$P = \rho\Omega_k^2 H^2 = 2.1\left(\frac{\dot{M}_{\mathrm{ac}}}{10^{-7}M_\odot/\mathrm{yr}}\right)\left(\frac{M_S}{M_\odot}\right)^{1/2}\left(\frac{R}{0.1\mathrm{AU}}\right)^{-5/2}\ \mathrm{erg/cm}^3$$

Here, $\dot{M}_{\mathrm{ac}}$ is the accretion rate, M_S is the stellar mass, P and ρ are the pressure and density, Ω_k is the frequency of the Kepler rotation, H is the thickness of the accretion rate.

Let us calculate the polarization of radiation of the accretion disk suggesting the magnetic field flux conservation. In this case $B_{\|} \sim R^{-2}$ and $B_{\perp} \sim R^{-1}$. We estimate the ratio:

$$P_{rel} = P_l(B, \mu)/P_l(\mu) \tag{11}$$

For $a = b = 4$, $P_{rel} = 0.3522$ and $\chi = 20.7°$. If we have $a = 8$ and $b = 2$, then $P_{rel} = 0.1279$ and $\chi = 41.2°$. In the opposite case, when $a = 2$ and $b = 8$, we obtain $P_{rel} = 0.1279$ and $\chi = 0.9°$.

The Eqs. (4) – (6) allow to derive the wavelength dependence of polarization if the radial dependence of the magnetic field components $B_{\|}$ and $B_{\perp}$ is known. This dependence can be derived in the framework of various accretion disk models. For example, for the standard Shakura–Sunyaev accretion disk, the radial dependence of the magnetic field in the standard accretion disk can be derived from the relation (Shakura & Sunyaev, 1973):

$$\frac{B^2}{8\pi} = \alpha \sqrt{P_{gas} P_{rad}} \tag{12}$$

where α is a well–known visibility parameter, introducing by Shakura & Sunyaev (1973).

4 Asymptotic Wavelength Dependence of Radiation Polarization on the Magnetized Accretion Disk

Let us present (in detail) the asymptotic formulae for polarization using the analytical calculations by Silant'ev (2002).

First of all, it is necessary to derive the Stokes parameters of the radiation from the accretion disk as a whole:

$$\begin{Bmatrix} F_I \\ F_Q \\ F_U \end{Bmatrix} = \int_{R_{in}}^{R_{out}} 2\pi r dr \mu \begin{Bmatrix} I \\ Q \\ U \end{Bmatrix} \tag{13}$$

where R_{in} and R_{out} are the disk boundaries. The disk is assumed to be axially symmetric. The degree of polarization must be calculated as

$$P_l = \frac{\sqrt{F_U^2 + F_Q^2}}{F_I} \tag{14}$$

The Stokes parameters I, Q and U are proportional to the radiation flux density at a given wavelength:

$$F_\lambda = A_\lambda \frac{1}{\exp \dfrac{2\pi hc}{k_B T \lambda} - 1} \tag{15}$$

If the temperature corresponding to the peak in the spectrum at the wavelength under consideration is observed far from the disk boundaries, i.e. if the radiation flux from boundary regions of the disk is low at this wavelength, then the integration limits in (12) can be assumed to be $R_{in} \to 0$ and $R_{out} \to \infty$. In this case, the polarization spectrum of the observed radiation acquires from (3)–(6) the following simple power law form (Gnedin et al., 2006):

$$P_l = \frac{0.09(1-\mu^2)}{J(\mu)\mu} C_\eta (15.3)^\eta \left(\frac{\lambda}{1\mu m}\right)^{(4\eta-6)/3} \left(\frac{\dot{M}_{ac}}{\dot{M}_{Ed}}\right)^{\eta/3} \left(\frac{B_{in}}{1G}\right)^{-1} \sim \lambda^{(4\eta-6)/3} \tag{16}$$

Table 2: The wavelength dependence of the radiation polarization for the various accretion disk models (Gnedin et al., 2006)

Model	$B_{eq}(R)$	$P_l(\lambda)$
Accretion disk with ion–supported flows	$\sim R^{-5/4}$	$\sim \lambda^{-1/3}$
Sunayev-Shakura disk Region (a), $P_r \gg P_g$	$\sim R^{-3/4}$	$\sim \lambda^{-1}$
Shakura-Sunayev disk (b) $P_g \gg P_r$	$\sim R^{-9/8}$	$\sim \lambda^{-1/2}$
Shakura-Sunayev disk (c) $P_g \gg P_r$	$\sim R^{-21/16}$	$\sim \lambda^{-1/4}$
Hot Accretion Disk with Plasma Viscosity	$\sim R^{-15/28}$	$\sim \lambda^{-9/7}$
Payne-Eardley Disk $P = P_g$, $\alpha = 1$	$\sim R^{-21/8}$	$\sim \lambda^{-1/8}$
Magnetic Accretion-Jet Ejection Disk without equipartition	$\sim R^{-5/2}$	$\sim \lambda^{4/3}$
Accretion disk with non-zero torque on its inner edge	$\sim R^{-15/16}$	$\sim \lambda^{-1}$
Disk with reprocessing	$\sim R^{-7/4}$	$\sim \lambda^{-1/8}$

where C_η is a constant that depends on the model parameter η:

$$C_\eta = \frac{\displaystyle\int_0^\infty \frac{x^{\eta+1}dx}{\exp x^{3/4} - 1}}{\displaystyle\int_0^\infty \frac{x\,dx}{\exp x^{3/4} - 1}} \tag{17}$$

Here we use the analytical expression for the polarization in an accretion disk without magnetic field, obtained by Silant'ev (2002). $J(\mu)$ is the function that describes the angular distribution of the outgoing radiation (Silant'ev, 2002).

Table 2 presents the wavelength dependence of polarization of radiation for various accretion disk models (Gnedin et al., 2006). The results of this Table imply that for various accretion disk models the wavelength dependence of the polarization is indicative of the pattern of radial dependence of the magnetic field in the disk.

5 Conclusions

We have developed a method that allows to estimate the magnetic field strength in the accretion disks surrounding the stars of various types. The polarization arises from scattering of the light emitted in a magnetized accretion disk. Due to Faraday rotation of the polarization plane, the resulting polarization degree differs essentially from the classical Rayleigh and Thomson cases. The main difference appears in the wavelength dependence of the polarization degree and position angle. Since the polarization spectrum of scattered radiation strongly depends on the accretion disk model, this allows to test various models of accretion disks around stellar objects with polarimetric observations. The obtained results allow us to calculate and to estimate the polarization parameters for a large variety of models of optically–thick magnetized accretion disks.

Acknowledgements. This research was supported by the program of the Presidium of RAS "Origin and Evolution of Stars and Galaxies", the program of the Department of Physical Sciences of the RAS "Extended Objects in the Universe", by the Federal Target Program "Scientific and scientific–pedagogical personnel of innovative Russia" 2009 – 2013 (GK 02.740.11.0246), and by the grant from the President of Russian Federation "The Basic Scientific Schools" (NSh–3645.2010.2).

References

Allen C. W., 1973, "Astronomical Quantities", Athlone, London
Chandrasekhar S., 1950, "Radiative Transfer", Clarendon press, Oxford
Dolginov A. Z., Gnedin Yu. N., Slinat'ev N. A., 1995, "Propagation and Polarization of Radiation in Cosmic Media", Gordon and Breach, New York
Gnedin Yu. N., Silant'ev N. A., 1997, "Basic Mechainsms of Light Polarization in Cosmic Media", Hartwood, Amsterdam
Gnedin Yu. N., Silant'ev N. A., Shternin P. S., 2006, Astron. Lett., 32, 39
Krasnopolsky R., Königl A., 2002, ApJ, 580, 987
Kurusz R. L., Peytremann E., Avrett E. H., 1974, "Blanketed Model Atmospheres for Early Type Stars", Smithonian Institute, Washington
Medvedev M. V., 2000, ApJ, 541, 811
Narayan R., Yi I., 1994, ApJ, 428, L13
Pariev V. I., Blackman E. G., Boldyrev S. A., 2003, A&A, 407, 403
Shakura N. I., Sunyaev R. A., 1973, A&A, 24, 337
Silant'ev N. A., 1993, ApJ, 419, 294
Silant'ev N.A., 2002, A&A, 383, 326
Silant'ev N. A., Piotrovich M. Yu., Gnedin Yu. N., Natsvlishvili T. M., 2009, A&A, 507, 171
Sobolev V. V., 1963, "A Treatise of Radiative Transfer", Van Nostrand, Toronto