S-stars motion around relativistic compact object Sgr A*.

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S-stars orbital parameters and SMBH SgrA*

We will use the parameters of Sgr A* from [1]:

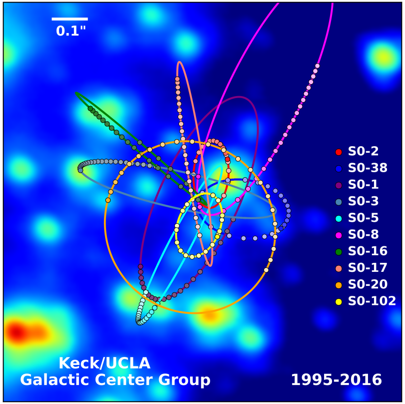
$$M = 4.28 \cdot 10^6 M_\odot$$

$$R_g = \frac{GM}{c^2} = 6.32 \cdot 10^{11} \text{ cm}$$

$$D = 8.32 \text{ Mpc}$$

Star	a, A.U.	e	T, yrs	v_p, km/s
S2 (S0-2)	1044	0.8839	16.0	7675
S38 (S0-38)	1178	0.8201	19.2	5709
S55 (S0-102)	897	0.7209	12.8	5108

Table: S-stars orbital parameters



Central 1.0×1.0 arcsecond of our Galaxy

S-stars astrophysical parameters

The S-star cluster was investigated by Eisenhauer et al. [2] in 2005. They have obtained some of the astrophysical parameters of S-stars:

- The magnitude of observed S-stars in K-band is $\sim 14^m \div 16^m$.
- The observed S-stars have *normal rotating velocities*, similar to solar neighborhood stars.
- The majority of S-stars appear to be a main sequence stars of $\mathrm{B0} \div \mathrm{B9}$ spectral classes.

S2 gravitational redshift and relativistic Doppler effect

Tuan Do et al., Relativistic redshift of the star S2 orbiting the Galactic center supermassive black hole, arXiv:1907.10731

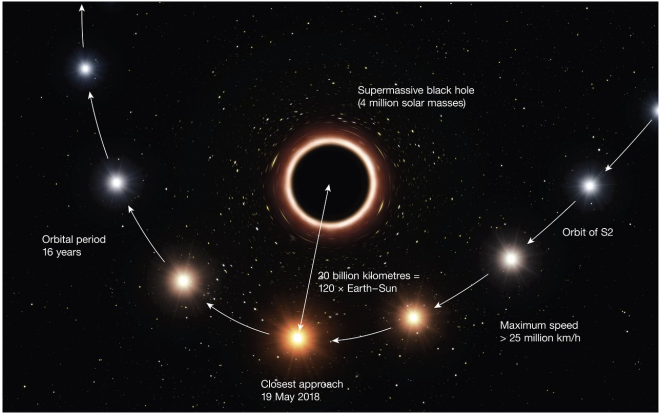


Image Credit: ESO/M. Kornmesser

G2 object - no tidal disruption

Elie Bouffard et al.,No sign of G2's encounter affecting Sgr A*'s X-ray flaring rate from Chandra observations, arXiv:1909.02175

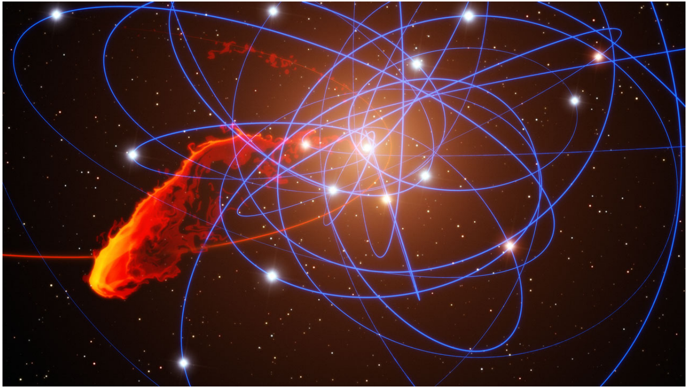


Image Credit: ESO/MPE/Marc Schartmann

Testing fundamental physics by observations of S-stars

Leor Barack et al., Black Holes, Gravitational Waves and Fundamental Physics: a roadmap, **arXiv:1806.05195**:
The purpose of this work is to present a concise, yet comprehensive overview of the state of the art in the relevant fields of research, summarize important open problems, and lay out a roadmap for future progress. This write-up is an initiative taken within the framework of the European Action on "Black holes, Gravitational waves and Fundamental Physics"
Astrophysical tests of the gravity physics: *"all involve gravity as a key component"*

Vesna Borka Jovanović et al., Constraining Scalar-Tensor gravity models by S2 star orbit around the Galactic Center, **arXiv:1904.05558**

*"The aim of our investigation is to derive a particular theory among the class of **scalar-tensor(ST) theories of gravity**, and then to test it by studying kinematics and dynamics of S-stars around supermassive black hole (BH) at Galactic Center (GC)"*

Testing fundamental physics by observations of S-stars

The work of Leor Barack et al. **arXiv:1806.05195** [4] actually has ∼200 authors:

Black holes, gravitational waves and fundamental physics: a roadmap

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Black holes, gravitational waves and fundamental physics: a roadmap 2

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Post-Newtonian Effects

We consider a possibility for Post-Newtonian measurement of the **energy density of the gravitational field** using orbital motion of S-stars.
The Post-Newtonian parameters are:

$$\frac{v}{c}, \quad \frac{v^2}{c^2}, \quad \frac{\varphi_N}{c^2}, \quad z_d, \quad z_g, \quad \varepsilon_g$$

v – orbital velocity

$\varphi_N = -GM/r$ – Newtonian potential

z_d – relativistic Doppler shift

z_g – gravitational redshift

ε_g – energy density of the gravitational field (erg/cm^3)

We consider the contribution of the energy density of the gravitational field to the observed value of the pericenter shift of the S stars (S2, S38, S55).

Equations of Motion (PN-GR)

In the frame of the geometrical gravity theory (GR), according to Brumberg ([3], 1991) the Lagrange function and corresponding Post-Newtonian equations of motion of a test particle in the gravitational field of central massive body is given by:

$$\frac{L}{m}=\frac{\dot{\mathbf{r}}^2}{2}\left(1+\frac{\dot{\mathbf{r}}^2}{4c^2}-(3-2\alpha)\frac{\varphi_N}{c^2}\right)-\varphi_N\left(1+(1-2\alpha)\frac{\varphi_N}{2c^2}+\alpha\frac{(\mathbf{r}\cdot\dot{\mathbf{r}})^2}{c^2r^2}\right)$$

$$\boxed{\ddot{\mathbf{r}}=-\boldsymbol{\nabla}\varphi_N\left(1+(4-2\alpha)\frac{\varphi_N}{c^2}+(1+\alpha)\frac{\dot{\mathbf{r}}^2}{c^2}-3\alpha\frac{(\mathbf{r}\cdot\dot{\mathbf{r}})^2}{c^2r^2}\right)+(4-2\alpha)\left(\boldsymbol{\nabla}\varphi_N\cdot\frac{\dot{\mathbf{r}}}{c}\right)\frac{\dot{\mathbf{r}}}{c}}$$

From the Lagrange function we also obtain energy and angular momentum:

$$\frac{E}{m}=\frac{\dot{\mathbf{r}}^2}{2}\left(1+\frac{\dot{\mathbf{r}}^2}{4c^2}-(3-2\alpha)\frac{\varphi_N}{c^2}\right)+\varphi_N\left(1+(1-2\alpha)\frac{\varphi_N}{2c^2}-\alpha\frac{(\mathbf{r}\cdot\dot{\mathbf{r}})^2}{c^2r^2}\right)$$

$$\frac{\mathbf{J}}{m}=[\mathbf{r}\times\dot{\mathbf{r}}]\left(1+\frac{\dot{\mathbf{r}}^2}{4c^2}-(3-2\alpha)\frac{\varphi_N}{c^2}\right)$$

Equations of Motion (PN-GR) in harmonic coordinates

Where $(\alpha = 1)$ for Schwarzschild coordinates, and $(\alpha = 0)$ for **harmonic** coordinates.
The equations of motion in harmonic coordinates:

$$\boxed{\ddot{\mathbf{r}} = -\boldsymbol{\nabla}\varphi_N\left(1 + 4\frac{\varphi_N}{c^2} + \frac{\dot{\mathbf{r}}^2}{c^2}\right) + 4\left(\boldsymbol{\nabla}\varphi_N \cdot \frac{\dot{\mathbf{r}}}{c}\right)\frac{\dot{\mathbf{r}}}{c}} \qquad (1)$$

The energy and the angular momentum in harmonic coordinates:

$$\frac{E}{m} = \frac{\dot{\mathbf{r}}^2}{2}\left(1 + \frac{\dot{\mathbf{r}}^2}{4c^2} - 3\frac{\varphi_N}{c^2}\right) + \varphi_N\left(1 + \frac{\varphi_N}{2c^2}\right)$$

$$\frac{\mathbf{J}}{m} = [\mathbf{r} \times \dot{\mathbf{r}}]\left(1 + \frac{\dot{\mathbf{r}}^2}{4c^2} - 3\frac{\varphi_N}{c^2}\right)$$

Pericenter shift (GR)

The Post-Newtonian equation of motion lead us into the effect of pericenter shift. The formula for the value of pericenter shift per one turn in GR is well known [3]:

$$\Delta\omega = \frac{6\pi R_g}{a(1-e^2)}$$

Dividing this value by *synodic period* T (the time that it takes for the probe body to go from pericenter to pericenter) gives us the value of the rate of the pericenter shift:

$$\dot{\omega} = \frac{6\pi R_g}{a(1-e^2)T} \tag{2}$$

Field Gravitation Theory

In the frame of the *field approach to gravitation*, formulated by Feynman in his famous "**Lectures on Gravitation**" [6], the gravitation is described as a *material field with positive energy density* in a flat Minkowski space (like all other fundamental physical interactions).

The gravitational symmetric tensor field has two irreducible parts: spin-2 attractive part and spin-0 repulsive part (V. V. Sokolov, Yu. V. Baryshev, [7], 1980). One of the main characteristics of the field is that it has energy-momentum tensor, and its 00-component (energy density of gravitational field) from [7] for static spherically symmetric field is given by:

$$\boxed{t_g^{00} = \frac{1}{8\pi G}(\boldsymbol{\nabla}\varphi_N)^2 \geqslant 0} \tag{3}$$

Problem of the energy-momentum pseudotensor in GR

According to Landau and Lifshitz "The Classical Theory of Fields" ([8], §96) the energy-momentum of the gravitational field in curved space cannot be defined in covariant form. It is so called the energy-momentum pseudotensor of gravitational field, and it depends on a choice of coordinates, so:
"it is meaningless to talk of whether or not there is gravitational energy at a given place".

Thus, the energy of gravitational field is **non-localizable** in the frame of geometrical approach for description of gravitation.

While in the Feynman's field approach to gravitation, the energy density of the gravitation field is a positive localizable physical quantity, e.g. Eq.(3).

Equations of motion (PN-FGT)

In the frame of the Field Gravitation Theory, according to Baryshev ([9], 1986) (see also G. Kalman, [10], 1961), the Lagrange function and corresponding Post-Newtonian equations of motion of a test particle in the gravitational field of central massive body is given by:

$$\frac{L}{m} = \frac{\dot{\mathbf{r}}^2}{2}\left(1 + \frac{\dot{\mathbf{r}}^2}{4c^2} - 3\frac{\varphi_N}{c^2}\right) - \varphi_N\left(1 + \frac{\varphi_N}{2c^2}\right)$$

$$\boxed{\ddot{\mathbf{r}} = -\boldsymbol{\nabla}\varphi_N\left(1 + 4\frac{\varphi_N}{c^2} + \frac{\dot{\mathbf{r}}^2}{c^2}\right) + 4\left(\boldsymbol{\nabla}\varphi_N \cdot \frac{\dot{\mathbf{r}}}{c}\right)\frac{\dot{\mathbf{r}}}{c}} \qquad (4)$$

We see that the Eq.(4) coincide with the equation Eq.(1) that we have obtained in the frame of harmonic coordinates of GR.

Pericenter shift (FGT)

From coincidence of equations of test particles motion Eq.(4) in FGT and Eq.(1) in GR it follows that the expression for the pericenter shift in FGT and GR is exactly the same.

The difference from GR is that the FGT formula has two different terms:

$$\dot{\omega} = \frac{6\pi R_g}{a(1-e^2)T} = \frac{7\pi R_g}{a(1-e^2)T} - \frac{\pi R_g}{a(1-e^2)T}$$

The first term with coefficient of 7π corresponds to the linear approximation, when in the field equations one does not take into account the non-linear contribution (energy of gravitational field itself).

The second term with coefficient of $-\pi$ occurs after taking into account the non-linearity due to the positive energy density of the gravitational field given by Eq.(3). This term corresponds to the measuring of the field energy of the gravity via pericenter shift observations.

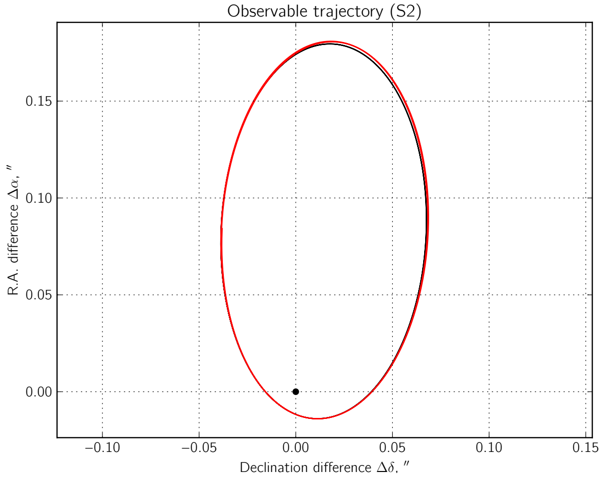
Pericenter shift values for S-stars

The pericenter shift values and parameter of $\frac{\varphi_N}{c^2}$ in pericenter predicted for S-stars are:

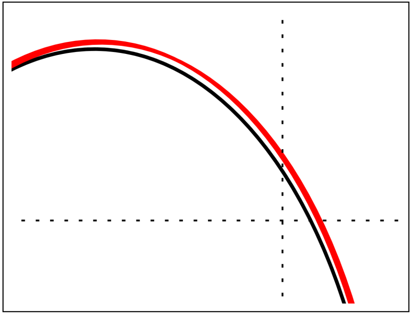
Star	S2	S38	S55
$\Delta\omega$	$12'$	$7.1'$	$6.7'$
$\dot{\omega}$	$45''/\mathrm{yr}$	$22''/\mathrm{yr}$	$31''/\mathrm{yr}$
$\dot{\omega} \cdot 100$ yrs	$1^\circ 15'$	$37'$	$52'$
$\left(\frac{\varphi_N}{c^2}\right)_{\mathrm{per}}$	$-3.48 \cdot 10^{-4}$	$-1.99 \cdot 10^{-4}$	$-1.69 \cdot 10^{-4}$

The observable effect is similar to one with **Mercury anomalous pericenter shift**, that Einstein explained. Now we can observe it with S-star cluster in the center of our Galaxy. For Mercury, the pericentral $\frac{\varphi_N}{c^2}$ is $-3.21 \cdot 10^{-8}$, and for binary pulsar PSR 1913+16 it is $-2.7 \cdot 10^{-6}$. We can see that the field at the pericenter of the S-stars orbits is stronger by 2 orders of magnitude. Although, it is still considered as a weak field.

S2 orbit PN effects: Orbit simulation



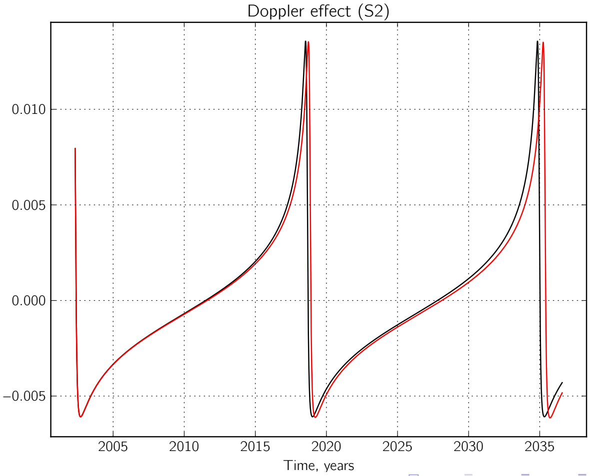
If we zoom in:



We can see that **Post-Newtonian** orbit is slightly thicker than **Newtonian**. That is caused by the pericenter shift effect.

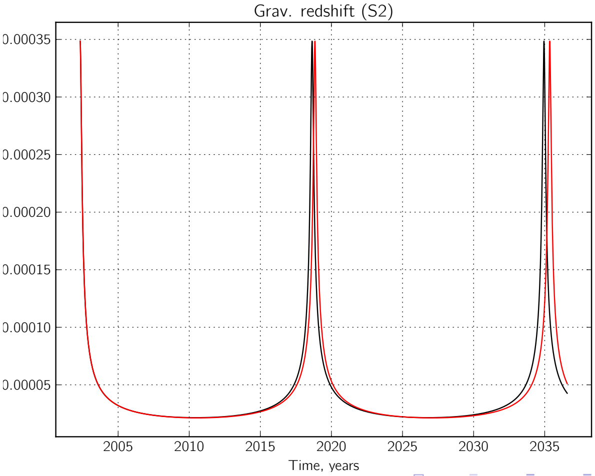
S2 orbit PN effects: Relativistic Doppler effect

The difference between the Newtonian and Post-Newtonian equations of motions leads into the small difference in the pericenter velocities, which leads into the small difference in orbital periods. We can see that after approximately 30 years of observations the difference is becoming larger due to time offset.



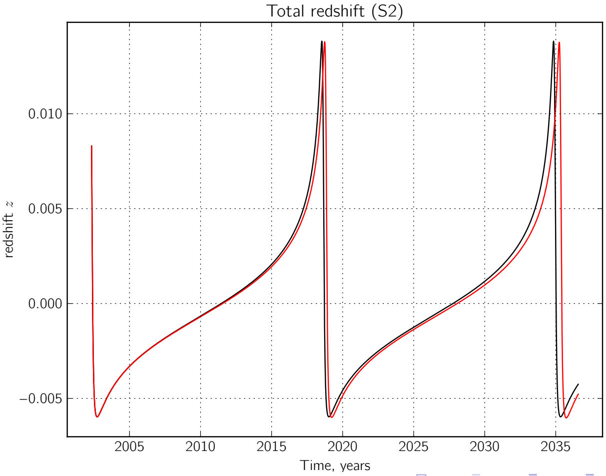
S2 orbit PN effects: Gravitational redshift

The same effect can be seen on the gravitational redshift plot.



S2 orbit PN effects: Total redshift

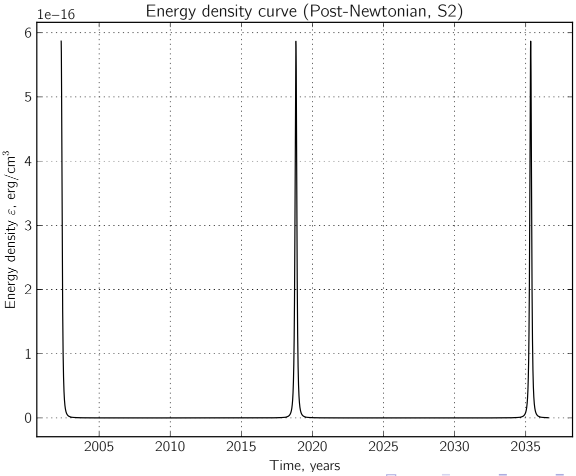
We obtained the total redshift plot, which takes into accound both Doppler shift and gravitational redshift.



S2 orbit PN effects: Energy density of the gravitational field

According to the Eq.(3) we also can measure the energy density of the gravitational field along the orbit of a star:

$$\varepsilon_g = \frac{(\nabla \varphi_N)^2}{8\pi G} = \frac{GM^2}{8\pi r^4}$$



Conclusion

The predicted for observations Post-Newtonian effects for the nearest to Sgr A* stars coincide in GR and FGT (Field Gravitation Theory).

In the frame of the FGT the effect of pericenter shift contains separate term, which corresponds to the direct contribution of the **positive localizable energy density** of the graviational field.

The value of positive energy density of gravitational field in the frame of field approach to gravitation is measurable. This conclusion is also consistent with the detection of positive energy of the gravitational waves by LIGO-Virgo antennas.

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