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Bispectral image reconstruction of the triple star ADS 11344

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Abstract. The calculation of the bispectrum of speckle interferograms allows image reconstruction of astronomical objects with diffraction resolution. The reconstruction procedure based on the calculation of a fraction of bispectrum is described, and its applicability to imaging of the triple star ADS 11344 is illustrated. The algorithm applies about 4 % of the total bispectrum, which makes it possible to diminish requirements to the storage capacity of a computer being used.

Key words: speckle interferometry – bispectrum – image reconstruction

1. Introduction

In classical speckle interferometry (Labeyrie, 1970) the computation of the image Fourier spectrum is accompanied by loss of information on phase, which results in unambiguous reconstruction of only the images of centrally symmetrical objects, such as a binary star with the companions of equal brightness. The phase information can be recovered from the bispectrum (BS) of speckle images (Lohmann et al., 1983; Lohmann and Wirnitzer, 1984; Bartelt et al., 1984). The determination of the total 4-dimensional BS from a series of many thousands of speckle images requires prolonged computation using powerful computers. The amount of computing time can be reduced considerably provided that only a fraction of BS is used, which appears sufficient for accurate estimates to be obtained (Orlov, 1994). Below the reconstruction of the triple star ADS 11344 image is described. When developing the algorithm, we took into account BS symmetries (Pehlemann, 1989), the reality of the image being reconsructed, and the limited size of the image Fourier spectrum.

2. Algorithm of bispectral image reconstruction

The proposed algorithm is intended for reconstruction of images of faint astronomical sources for which a separate speckle interferogram registers a relatively small number of photon impacts and may be written as the sum of delta functions:

$$d(\vec{x}) = \sum_{j=1}^{N} \delta(\vec{x} - \vec{x}_j), \qquad (1)$$

where $\delta(\vec{x})$ is the Dirac delta function, $\vec{x_j}$ is the two-dimensional radius-vector of the j-th photon, N is the number of events recorded in a frame. The phase is known to contain more information about the image than the module. Wirnitzer (1985) showed that for the reconstruction of a phase spectrum the BS analysis seems most advantageous since it provides a maximum signal to noise ratio (S/N). The procedure of image reconstruction comprises 5 stages:

- BS calculation of an individual speckle image; only part of BS selected with allowance for 12 symmetries and frequency cut-off of the aperture is used (Orlov, 1994);
- averaging of the BS over all images;
- phase reconstruction of the image Fourier spectrum;
- calculation of the module by the Labeyrie technique (Labeyrie,1970);
- phase-module combination for obtaining the image.

At the first stage the BS elements are calculated using Fourier transform. In the discrete form the BS may be written as

$$D^{(3)}(i,j,k,l) = D(i,j) \cdot D(k,l) \cdot D(-i-k,-j-l), \qquad (2)$$

where $D^{(3)}(i,j,k,l)$ is the BS value at the point (i,j,k,l), $D(i,j)$ is the image Fourier spectrum value at the point (i,j). To allow for the photon bias resulting from correlation of photons with each other (Goodman and Belsher,1976), it is necessary to compute the function:

$$s(\vec{x}) = \sum_{k=1}^{N} \sum_{j=1}^{N} \delta(\vec{x} - \vec{x_j} + \vec{x_k}), \qquad (3)$$

whose Fourier transform (FT) is a function of BS bias. The unbiased BS estimate is

$$I^{(3)}(i,j,k,l) = D^{(3)}(i,j,k,l) - S(i,j) \qquad (4)$$
$$-S(k,l) - S(-i-k,-j-l) + 2 \cdot N,$$

where $S(i,j)$ is the bias value at the point (i,j). The reconstruction of the image phase spectrum from the BS phase is based on the following relationship:

$$exp\{\imath[\varphi(i,j) + \varphi(k,l) - \varphi(i+k,j+l)]\} = \qquad (5)$$
$$exp\{\imath\beta(i,j,k,l)\},$$

where $\imath$ is the square root of -1, $\varphi(i,j)$ is the image phase spectrum, $\beta(i,j,k,l)$ is the BS phase. Most frequently the phase is reconstructed by recursive scheme (Hofmann and Weigelt 1986). For phase reconstruction from BS we applied the least squares method that allows the error at high spatial frquencies to be avoided and the signal-to-noise ratio at low frequencies improved. To do this, we minimize the functional:

$$\sum_i \sum_j \sum_k \sum_l \{[\varphi(i,j) + \varphi(k,l) \qquad (6)$$
$$-\varphi(i+k,j+l) - \beta(i,j,k,l)]/\sigma_\beta(i,j,k,l)\}^2,$$

where $\sigma_\beta(i,j,k,l)$ is the standard bias of the BS phase. The value of this expression is chosen as the criterion of convergence as well. The spectrum points with a module value close to zero are taken up separately since the phase is not defined for them. When averaging, a zero weight is assigned to these points. As a rule, after 10-15 iterations the value of expression (6) changes only slightly.

To reconstruct the Fourier module of the object image, we applied the technique of Labeyrie (1970), which may be realized in both the Fourier region and the spatial plane. In the former case the averaged power spectrum $< |I_n(\vec{u})|^2 >$ of speckle interferograms is computed, in the latter - the autocorrelation $< \imath_m^{(2)}(\vec{x}) >$. If the mean number of photon events in the speckle images is less than the number of pixels of the detector matrix, then it is more efficient to calculate the averaged autocorrelation function (AF):

$$< \imath_m^{(2)}(\vec{x}) >=< \imath_n(\vec{x}) \otimes \imath_n(\vec{x}) >= \qquad (7)$$
$$< \textstyle\sum_{i=1}^N \sum_{j=1;j\neq i}^N \delta(\vec{x} - \vec{x_j} + \vec{x_i}) > .$$

The condition $j \neq i$ should be satisfied to remove photon bias. In the computation care should be taken that the vectors going beyond the reference region on one side add on the other. Otherwise the obtained AF will be affected by the detector window function.

Since the Fourier spectrum components are reconstructed by independent methods, the connection between module and phase is broken. This leads to the fact that the inverse FT yields a distorted object image. To restore the connection we use the iterative algorithm. The distorting operator in this case is chosen identically equal to zero. As the restrictions we use the positivity and the finite size of the image and its AF. The procedure includes the following steps:

- reverse FT of the obtained spectrum is computed:

$$O(\vec{u}) = |O(\vec{u})| exp\{\imath\varphi(\vec{u})\}, \qquad (8)$$

where $|O(\vec{u})|$ is the Fourier module of the object image;

- all the image elements outside the region of determination are put equal to zero;
- the FT is computed;
- all the elements of the image Fourier spectrum outside the region limited by the telescope frequency cut-off are forced to zero;
- all the elements having negative values and all the elements beyond the region of determination in the AF, obtained as an inverse FT of the squared module, are forced to zero;
- the inverse FT of the AF, a new squared module of the object image is obtained;
- the Fourier spectrum module is replaced by a new one.

The iterations are repeated until the value of the selected criterion is stabilized.

3. Observations of ADS 11344 and data processing

ADS 11344 (HD 170109) is a triple system of the spectral type $K0$V and the magnitude $m_v = 7.15$ (Echevarria et al, 1979). The wide pair AC (STT 351) was discovered by O. Struve in 1846. Apparently the measurements of O. Struve were affected by the presence of a third companion since the angular distance determined by him, $\rho \approx 0.5''$, is in poor agreement with recent obsevations. The existence of another companion in the system STT 351 was established by Hussy 50 years later using a 36-inch refractor. The new pair named Hu 66 shows a slow reverse orbital motion with a period of about 500 years. The data on the brightness difference between the companions have been taken mainly from visual observations and therefore are very conflicting. The system has been observed regularly with large telescopes using the speckle interferometric technique for the last few years. The small magnitude difference between the companions and comparable angular distances make this object suitable for the methods of image reconstruction to be tested.

We conducted speckle interferometric observations of ADS 11344 in August 1991 at the SAO 1 m telescope using a television photon counting system (Balega et al., 1990). An interference filter $\lambda/\Delta\lambda$=600/15 nm and microobjective providing a

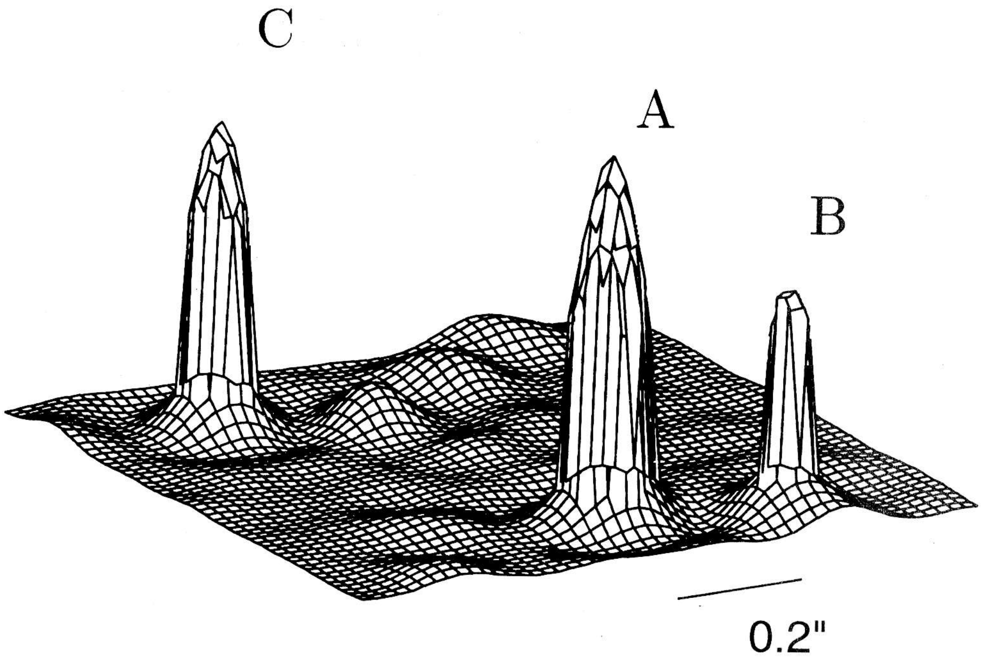

Figure 1: *Image of ADS 11344 reconstructed from the bispectrum of speckle images.*

scale of 0.011″/pixel in the plane of the photocathode were employed. The angular size of the field was 2.8″. With a seeing $\lambda/r_o \simeq 3''$ we had $(D/r_o)^2 \approx 500$ speckles per image. Here r_o is the Fried atmospheric parameter (Fried, 1966). The image exposure time was 20 ms. For 10 minutes of observations in the photon counting mode with a mean flux of 150 events per frame, 30000 speckle images were stored on magnetic tape. The same number of images was recorded for a reference single star.

The initial reduction made it possible to systematize the data with due regard for atmospheric conditions, guiding quality during the exposure, and to correct for distortion introduced by the detector. The preliminary reduction of speckle images comprised the following steps:

- removal of traces caused by the remnants of photons and masking of permanent detector defects;
- selection of images by the number of photon events per frame and by the size of the region of photons scatter;
- correction of distortions resulting from the limited dynamic range of the television system by adding events to the recorded images.

The size of a photon spot detected by the television camera depends on a number of factors and practically always exceeds the pixel size. As a result,
the system is unable to record close pairs of event
and the AF centre degenerates into zero values. T
BS, which is impossible to be corrected at all spati
frequencies, is subject to still stronger distortions.
solve this problem, different techniques are used. O
of them is the splitting of the light flux into two a
using of separate detectors to record the two ima
(Thiebaut, 1993). In this case analysis is based
the computation of cross-correlation of speckle ima
recorded simultaneously with different detectors. A
other procedure consists in analyzing the profile o
photon event and allowance made for the correspo
ing bias in the BS (Hofmann, 1992). We propose
compensate for the distortions by adding photons
each frame. Accurate coordinates of the photons, l
as a result of their sticking together, are impossi
to define, however one can find their most proba
locations. To do this, one has to determine the me
profile of photon event, using the following algorith
Since the Fourier spectrum of the AF is the pow
spectrum of the image, it must be a real non-negati
function, and therefore the inverse FT from the ne
tive part of the Fourier spectrum of the obtained
yields an inverted AF of the mean profile of pho
events. Assuming that the profile can not be asy
metric, it can easily be obtained from the AF. T
probability density function of the lost photons is c
sen to be proportional to the difference between

speckle image, convoluted with the profile of photon events, and the initial image. The probability density is normalized by the number of photons to be added to the image. This number does not generally exceed 3-4 % of the number of events in the frame. Using a random value with computed probability density the coordinates of the missing events are simulated, which are introduced in the image. The described procedure allows partial overcoming of the difficulties of phase reconstruction from the BS, which result from the nonlinearity of the detector counting characteristics.

Since the Fourier spectrum of the object lies inside a circle with a radius of 13 elements, the total BS contains about 400000 complex elements. The account of the 12 symmetries and rejection of the BS elements that do not contain phase information, reduces the amount of computations dozens of times, preserving all information about the object. For the spectrum reconstruction we used the part of the BS for which the vectors (i, j) and (k, l) are located simultaneously in the same quadrant. We determined the bias function $S(i, j)$ necessary for obtaining the unbiased BS estimate. The reduction time of a single speckle image depends on the number of photons and takes for our data about 10 seconds at AT-486 computer. The BS storage over all speckle images of ADS 11344 obtained took 72 hours of continuous counting. The rest of the processing, which includes the restoration of phase spectrum, reconstruction of image from Fourier components and filtering, requires considerably less time (3-5 minutes per iteration).

In Fig.1 an image of ADS 11344 reconstructed from the BS is presented. Only the central part of the image, 64×64 $(0.7'' \times 0.7'')$ elements in size, is shown. Position angles for the system are: $\Theta_{AB} = 248° \pm 3°, \Theta_{AC} = 20° \pm 2°$; angular distances: $\rho_{AB} = 0.299'' \pm 0.005'', \rho_{AC} = 0.728'' \pm 0.005''$. The high-frequency spatial information in the reconstructed image is limited by the difraction cut-off $\lambda/D = 0.12''$ of the 1 m aperture at 600 nm. The magnitude differences between the components are: $m_A - m_B = -0.52 \pm 0.05, m_A - m_C = 0.13 \pm 0.05$. The principal source of error in determining relative brightness difference is the procedure of adding photons to individual speckle images for suppressing the effect of photon loss. It is possible that the former definition of the A star as the primary is caused by the variability of one or both stars in the system Hu 66AB (Baize, 1983).

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