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GRAVITATIONAL LENSES AND CATASTROPHE THEORY

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ABSTRACT. *A brief review of the main results and ideas of catastrophe theory are given. The relation of these results with the stable images in gravitational lenses and caustics is discussed.*

An important part of an analysis of observational data of astronomical objects is examination of structurally stable and unstable images. The importance of this examination is conditioned by the fact that a lot of small parameters are not taken into theoretical consideration. The example of a structurally unstable image of a point source is the image of Einstein ring in the focal line, since if an observer is near the focal line then he sees three images instead of Einstein ring. The caustics of the ray system play an important role in the analysis of images in gravitational lenses. The classification of stable caustic surfaces is given in (Arnol'd, 1976). G. Airy, T. Percey, and V. Arnol'd calculated the intensity near caustics surfaces. The intensity near a singularity of fold type is defined by Airy integral $I \sim 2k^{1/6}A_1(ak^{2/3})$, where $k=2\pi/\lambda$,

$$2A_1(t)=\int_R e^{(x^3+tx)}dx,$$

and intensity near a singularity of cusp type is defined by Persey integral (Gilmore, 1981)

$$Pe(z,u)=\int_R exp[i(\beta^4/8-(\beta^2/2)+u\beta)]d\beta.$$

The intensity near other singularities is given in (Arnol'd, 1976) ($I \sim k^d$, d=1/6 for fold (A_2), d=1/4 for cusp (A_3), d=3/10 for swallow-tail (A_4), d=1/3 for pyramyd

and purse (D_4)). It is known that there are the following stable singularities in three-dimensional space: swallow tail, pyramyd, and purse (codim=3). There are stable singularities in four-dimensional spaces: butterfly (A_5) and parabolic umbilic (D_5) (codim=4). The A_μ group is the (μ+1)-degree polynomial roots (sum of these roots equals 0) of transpositions group. More correctly, the common position caustic in ($n \leqslant 5$)-dimensional space is stable and R-equivalent in any point neighbourhood to the Cartesian product of A_μ, D_μ, E_μ ($\mu - 1 \leqslant n$) and non-singularity manifold or to the set of these transversal caustics (Arnol'd & Givental', 1985; Arnol'd et al., 1982).

These two singularities (A_5 and D_5) may exist as moving caustic surfaces in three-dimensional space. The classification of the stable singularities and their metamorphoses is given with the application of Coxeter group and Dynkin diagrams (Arnol'd, 1976). If the observer moves near these singularities, then when he is located on corresponding caustic surfaces he sees the increase of brightness that is associated with these singularities, and the increase is larger than the brightness increase for caustics of fold and cusp types. All stable caustic surfaces and brightness near the corresponding caustic surfaces are given in (Arnol'd, 1983).

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